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ON THE DETERMINATION OF OPTIMUM COMMAND ADDRESS CODES

BY

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GREENBELT, MARYLAND

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SUMMARY

In order to minimize the probability of a spacecraft being addressed erroneously by the transmitted command address code of another spacecraft, an analysis has been conducted on the determination of optimum command address codes. The criteria used in this analysis for determining optimum command address codes are: (1) the Hamming distance; and (2) the code subsequences (i.e., no subsequence of a command address code can be identical to another command address code).*

In general, for a given word length, the aforementioned probability of error decreases as the Hamming distance increases. For two or more word lengths, the probability of error also decreases as the code subsequences (of each command address code), which are identical to command codes, decrease. It is shown and/or proven that the number of command address codes is reduced as: (1) the Hamming distance increases; and/or (2) the code subsequences, which are identical to command address codes, decrease. The reduction in the number of command address codes is especially severe when the code subsequences criterion is satisfied (i.e., no subsequence of a command address code is identical to any other command address code). This severe reduction in the number of command address codes makes it impractical to use a set of command address codes of more than two distinct word lengths. A set of command address codes of one (i.e., the same) word length is ideal since the code subsequences criterion is no longer applicable.

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* It is understood that other criteria must be considered in the determination of optimum command address codes. Further analysis is being presently conducted by the author as well as other GSFC personnel.

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INTRODUCTION

At the request of E. Melendey of the Command and Control Section, Space Data Control Branch, an analysis has been conducted to determine optimum command address codes which can be assigned to NASA spacecrafts by the GSFC Frequency Control Officer. The optimum command address codes were to be determined or selected from the sets of all 4, 5, ... , 31, and 32-digit binary words in a manner to minimize the probability of a spacecraft being addressed erroneously by the transmitted command address code of another spacecraft. The purpose of this write-up is to report the results of the aforementioned analysis which could revolutionize the present GSFC method(s) for assigning command address codes to NASA spacecraft.

Several optimum sets of 4-digit and 8-digit command address codes which respectively have a minimum mutual Hamming distance of 2 and 4 are given in Table I.

The upper bound for the number of binary words or codes of length n, which have a mimimum mutual Hamming distance of d, are given in Table II for $n = 4, 5, \ldots, 13$ and $d = 2, 3, \ldots, 13$.

BINARY CODE GEOMETRY

Geometrically, every binary code word, having n-digits, can be represented as a point in the n-dimensional space with every coordinate being either 0 or 1. More specifically, every binary code word, having n digits, can be represented as a vertex of a unit n-dimensional cube. As a matter of fact, there is a one-to-one correspondence or mapping of the set of all n-digit binary words <u>onto</u> the set of all vertices of the unit n-dimensional cube. It is easy to prove that there are 2^n members in the set of all vertices of the unit n-dimensional cube.[1] Hence, it follows from above that there are 2^n members in the set of all n-digit binary words.

DETERMINATION CRITERIA

The criteria for determining a set of optimum command address codes from the sets of all 4, 5, ... , 31, and 32-digit binary words are:

(1) the Hamming distance, if the codes have the same word length (i.e., the same number of binary digits in each code).

(2) the code subsequences, if the codes have distinct word lengths. (That is, does the code sequence of binary digits contain a subsequence of binary digits which is identical to a code of smaller word length ?).

Let $A = (a_1, a_2, \ldots, a_n)$ and $B = (b_1, b_2, \ldots, b_n)$ be binary words (i.e., $a_i = 0$ or 1, and $b_j = 0$ or 1 where $i, j = 1, 2, \ldots, n$). The Hamming distance between A and B is defined to be the number of coordinates for which A and B are different. For example, if A = 00110 and B = 10101, the Hamming distance is 3 since A and B are different in the first, fourth, and fifth coordinates. The Hamming distance may also be defined as the least number of edges, on the unit n-dimensional cube, which must be traversed in order to go from A to B.

ON THE OPTIMUM SETS OF COMMAND ADDRESS CODES

A FAP computer program has been written for the IBM 7090/7094 to find the optimum set of n-digit command address codes (including the code whose coordinates are all zeros) which have a minimum mutual Hamming distance of d, and whose total number (i.e., the total number of members in the optimum set) constitute the maximum number of n-digit binary words which have a minimum mutual Hamming distance of d. The maximum number of n-digit binary words which have a minimum mutual Hamming distance of d is often referred to as the upper bound for the number of n-digit binary words which have a minimum mutual Hamming distance of d. The upper bound will be denoted by B(n, d).

The set of n-digit binary words, as determined by the FAP computer program, is not the only set of n-digit binary words which have a minimum mutual Hamming distance of d, and whose total number is equivalent to B(n, d), unless d = 1, in which case the FAP computer program is not needed since $B(n, 1) = 2^n$. That is, the members of the set of all n-digit binary words (i.e., the set of all vertices of the unit n-dimensional cube) have a minimum mutual Hamming distance of 1. When $d > 1$ (note that d cannot exceed n), the total number of sets of n-digit binary words which have a minimum mutual Hamming distance of d, with the total number of members in each set equivalent to B(n, d), can be expressed as

$$\frac{2^n}{B(n, d)}$$

The $\left((2^n/B(n, d)) - 1\right)$ "other" sets can be easily generated from the "computed" set (i.e., the set which is determined by the FAP computer program) by changing 0 to 1 or 1 to 0 for one or more coordinates or digit positions in each n-digit binary word of the "computed" set. For example, suppose that $n = 3$ and $d = 3$, then $B(n, d) = B(3, 3) = 2$ and the "computed" set is

000
111

Since $2^n/B(n, d) = 2^3/B(3, 3) = 4$, there are $(2^n/B(n, d)) - 1 = (2^3/B(3, 3)) - 1 = 3$ "other" sets which can be generated from the "computed" set. Generating these sets, we have

001	010		100
110,	101,	and	011

For the left-most set, the third coordinate or digit position was complemented (i.e., 1 to 0 or 0 to 1) in each of the 3-digit binary words of the "computed" set. For the second set from the left, the second coordinate was complemented. Finally, for the third set from the left, the first coordinate was complemented.

Observe, in the above example, that the Hamming distance between the first word in each of the "other" sets and the first word in the "computed" set is 1. This observation is useful when one has to determine which coordinate(s) or digit position(s) to complement in the "computed" set in order to generate the "other" set(s). In some generations or cases, a set of the "other" sets will not contain a word which has a Hamming distance of 1 between itself and the <u>first word of the "computed" set</u> (i.e., the word whose coordinates are all zeros). However, this will be easily recognized. In any event, <u>every</u> word in each of the "other" set(s) <u>must</u> have a Hamming distance of $d' < d$ between itself and at least one word in the "computed" set.

MINIMIZING THE PROBABILITY OF ERROR OF THE COMMAND ADDRESS CODES

The minimum mutual Hamming distance between the codes or words of a set of n-digit binary words is closely related to the probability of error of that set (i.e., the probability that a transmitted n-digit word of the set will be erroneously received as another n-digit word of the set). In general, the probability of error of a set of n-digit binary words decreases as the minimum mutual Hamming distance of the members of the set increases. However, by increasing the minimum mutual Hamming distance, the number of codes or words in the set of n-digit binary words will, in most cases, be decreased (i.e., the number of words in a set of n-digit binary words which have a minimum mutual Hamming distance of d_1 will, in most cases, be smaller (and never larger) than the number of words in a set of n-digit binary words which have a minimum mutual Hamming distance of d_2, if $d_1 > d_2$). In other words, the larger the minimum Hamming distance, the smaller the number of vertices on the unit n-dimensional cube which can mutually satisfy that minimum Hamming distance. Expressing the above statements mathematically, we have:

$$\lim_{d \to n} B(n, d) = 2$$

and

$$\lim_{d \to n+1} B(n, d) = 0$$

but

$$\lim_{d \to 1} B(n, d) = 2^n$$

Since both $B(n,d)$ and the probability of error of the set of n-digit binary words or command address codes, which have a minimum mutual Hamming distance of d, are decreased as d (the Hamming distance) is increased, it becomes necessary to have a "trade-off" between the minimization of the probability of error and the number of n-digit words required for command address codes.

In the work request by E. Melendey, the Hamming distance (between codes of the same length) was not specified. However, it was indicated that the number of words or codes, which were required for command address codes, in each of the sets of 4, 5, ... , 31, and 32-digit binary words was relatively small. Furthermore, it was requested that: no member of the set of 4-digit command address codes be the same as <u>any</u> 4-digit subsequence of any member of the set of 5-digit command address codes; no member of the sets of 4 and 5-digit command address codes be the same as <u>any</u> 4 or 5-digit subsequence of any member of the set of 6-digit command address codes; etc. . . . .
In short, for $m < n$, no member of the set of m-digit command address codes should be the same as any m-digit subsequence of any member of the set of n-digit command address codes.

In order to satisfy the Hamming distance and "trade-off" criteria, the author decided to let

$$d = \frac{n}{2}, \text{ truncated to the nearest integer.}$$

The upper bound B(n, d) for the number of members of each of the sets of 4, 5, ... , 25, and 26-digit binary codes, which have the aforementioned Hamming distance (i.e., $d = n/2$, truncated to the nearest integer), was determined by means of the computer program as follows:

n	B(n, d)
4	8
5	16
6	8
7	16
8	16
9	16
10	8

n	B(n, d)
11	16
12	16
13	16
14	16
15	32
16	32
17	32

n	B(n, d)
18	8
19	16
20	16
21	32
22	16
23	32
24	32

n	B(n, d)
25	32
26	16

The code subsequences criterion (i.e., the request that, for $m < n$, no member of the set of m-digit command address codes should be the same as any m-digit subsequence of any member of the set of n-digit command address codes) can be satisfied only at the expense of severely limiting the number of usable command address codes. We proceed to show that this (i.e., the statement that the code subsequences criterion can be satisfied only at the expense of severely limiting the number of usable command address codes) is true.

Limitations of the Code Subsequences Criterion on the Number of Usable Command Address Codes

As a first step toward showing that the code subsequences criterion can be satisfied only at the expense of severely limiting the number of usable command address codes, we prove that if: (1) every member of the set of all j-digit binary words ($j = 1, 2, 3, \ldots, n$) were used as a command address code; and (2) the code subsequences criterion were satisfied, every binary word, whose word length (i.e., the number of digits in the word) is greater than j, is unusable as a command address code.

Proof:

Consider the set

$$S = \{00, 01, 10, 11\}$$

of all 2-digit binary words (i.e., the set of all vertices of the unit 2-dimensional cube). Also, consider the set

$$T = \{000, 001, 010, 011, 100, 101, 110, 111\}$$

of all 3-digit binary words (i.e., the set of all vertices of the unit 3-dimensional cube). The set T can be rewritten as

$$T = \{000, 010, 100, 110\} \cup \{001, 011, 101, 111\}$$

Now, observe that each of the above two subsets of T is merely an "extension" of the set S. That is, the set $\{000, 010, 100, 110\}$ can be easily generated from the set S by "attaching" or placing the digit "0" to the right of the two digits of each member of the set S. Similarly, the set $\{001, 011, 101, 111\}$ can be easily generated from the set S by "attaching" the digit "1" to the right of the two digits of each member of the set S. Hence, the set T can be easily generated from the set S by "attaching" the digit 0 to the right of the two digits of each member of S (for half of the members of T), and then "attaching" the digit 1 to the right of the two digits of each member of S (for the other half of the members of T).

Similar to above, the set

$$W = \{0000, 0001, 0010, 0011, 0100, 0101, 0110, 0111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111\}$$

$$= \{0000, 0010, 0100, 0110, 1000, 1010, 1100, 1110\} \cup \{0001, 0011, 0101, 0111, 1001, 1011, 1101, 1111\}$$

of all 4-digit binary words (i.e., the set of all vertices of the unit 4-dimensional cube) can be easily generated from the set T by "attaching" the digit 0 to the right of the three digits of each member of T (for half of the members of W), and then "attaching" the digit 1 to the right of the three digits of each member of T (for the other half of the members of W).

Again, similar to above, the set of all 5-digit binary words can be easily generated from the set W or the set of all 4-digit binary words; the set of all 6-digit binary words can be easily generated from the set of all 5-digit binary words; . . . .; the set of all n-digit binary words can be easily generated from the set of all $(n-1)$-digit binary words.

From the above results, we see that every member of the set of all 3-digit binary words contains a 2-digit subsequence which is identical to at least one member of the set of all 2-digit binary words. Similarly, every member of the set of all 4-digit binary words contains a 3-digit subsequence which is identical to at least one member of the set of all 3-digit binary words. But, since every member of the set of all 3-digit binary words contains a 2-digit subsequence which is identical to at least one member of the set of all 2-digit binary words, it follows that every member of the set of all 4-digit binary words also contains a 2-digit subsequence which is identical to at least one member of the set of all 2-digit binary words. We could continue the above reasoning to show that: every member of the set of all 5-digit binary words contains 4-digit, 3-digit, and 2-digit subsequences which are respectively identical to at least one member of the sets of all 4-digit, 3-digit, and 2-digit binary words; every member of the set of all 6-digit binary words contains 5-digit, 4-digit, 3-digit, and 2-digit subsequences which are respectively identical to at least one member of the sets of all 5-digit, 4-digit, 3-digit, and 2-digit binary words; . . . .; and every member of the set of all n-digit binary words contains (n -1)-digit, (n - 2)-digit, ... , 3-digit, and 2-digit subsequences which are respectively identical to at least one member of the sets of all (n - 1)-digit, (n - 2)-digit, ... , 3 - digit, and 2-digit binary words. Furthermore, for $i = 2, 3, 4, \ldots, n$, it is trivial that every member of the set of all i-digit binary words contains a digit which is identical to at least one member of the set of all 1-digit binary words (i.e., the set $\{0, 1\}$). Hence, for $j = 1, 2, 3 \ldots, n$, it follows that every member of the set of all j-digit binary words contains (j - 1)-digit, (j - 2)-digit, ... , 3-digit, and 2-digit subsequences, and <u>a digit</u> which are respectively identical to at least one member of the sets of all (j - 1)-digit, (j - 2)-digit, ... , 2-digit, and <u>1-digit</u> binary words.

From the above results, it now follows that if: (1) for $j = 1, 2, \ldots, n$, every member of the set of all j-digit binary words were used as a command address code; and (2) the code subsequences criterion were satisfied, every member of the sets of all (j + 1)-digit, (j + 2)-digit, (j + 3)-digit, ... , and etc. binary words would be <u>unusable</u> as a command address code since it contains at least one <u>used</u> command address code of shorter word length (i.e., j) than itself. In other words, every binary word, whose word length is greater than j, is <u>unusable</u> as a command address code.

Q.E.D.

The argument of the above proof can also be utilized to easily show or prove that if: (1) every member of the set of all j-digit binary words ($j = 1, 2, \ldots, n$) were used as command address codes; and (2) the code subsequences criterion were satisfied, every binary word, whose word length is <u>smaller</u> than j, <u>could not</u> be used as a command address code. The proof is left to the reader.

From the above results, it is clear that the minimum mutual Hamming distance of the selected set of j-digit command address codes ($j = 1, 2, \ldots, n$) can <u>not</u> be 1, since $B(j, 1) = 2^j$ (i.e., the selected set of j-digit command address codes would be equivalent to the set of all j-digit binary words, which mean that every binary word whose word length <u>is not</u> j is unusable as a command address code). Hence, the minimum mutual Hamming distance of the selected set of j-digit command address codes must be 2 or more. If the minimum mutual Hamming distance of the selected set of j-digit command address codes is 2, then the set of j-digit command address codes contains one-half of the members of the set of all j-digit binary words since $B(j, 2) = 2^{j-1}$.

Again, consider the set

$$S = \{00, 01, 10, 11\}$$

of all 2-digit binary words. The set S can be expressed as

$$S = \{00, 11\} \cup \{01, 10\}$$

where the two subsets of S are the two sets of all 2-digit binary words which have, for each set, a minimum mutual Hamming distance of 2. It was proven above that we can not use both subsets of S as command address codes. Hence, as a first choice, we select the subset $\{00, 11\}$ as the set of 2-digit command address codes. Now, again, consider the set

$$T = \{000, 001, 010, 011, 100, 101, 110, 111\}$$

Since 00 and 11 are command address codes, the set of 3-digit command address codes is $\{010, 101\}$ after the code subsequences criterion is satisfied. (Note that the set of 3-digit command address codes is $\{000, 111\}$ if the subset $\{01, 10\}$ is used as the set of 2-digit command address codes).

From the set of all 4-digit binary words, W, we see, since 00, 11, 010 and 101 are selected command address codes, that the set of 4-digit command address codes is empty (i.e., no elements or members in the set) after the code subsequences criterion is satisfied. (Note that the set of 4-digit command address codes is also empty if 01, 10, 000, and 111 are the selected command address codes). It now follows that the sets of 5-digit, 6-digit, ... , (n - 1)-digit, and n-digit command address codes are empty since every member of each of the aforementioned sets contains a 4-digit subsequence which is identical to at least one member of the set of all 4-digit binary words.

Suppose we first select command address codes from the set of all 4-digit binary words, W. As above, we can not use every member in the set W as a command address code. Hence, similar to above, we select a set of 4-digit binary words, which have a minimum mutual Hamming distance of 2, as the set of 4-digit command address codes. Since the set W can be expressed as

$$W = \{0000, 0011, 0101, 0110, 1001, 1010, 1100, 1111\} \cup \{0001, 0010, 0100, 0111, 1000, 1011, 1101, 1110\}$$

(where the two subsets of W are the two sets of all 4-digit binary words which have, for each set, a minimum mutual Hamming distance of 2), we select the subset {0000, 0011, 0101, 0110, 1001, 1010, 1100, 1111} as the set of 4-digit command address codes. Now, since 0000, 0011, 0101, 0110, 1001, 1010, 1100, and 1111 are selected command address codes, the set of 5-digit command address codes, after the code subsequences criterion is satisfied, is {00010, 00100, 01000, 01110, 10001, 10111, 11011, 11101} whose members have a minimum mutual Hamming distance of 2. But, from the set of all 6-digit binary words, we find that the set of 6-digit command address codes is empty, after the code subsequences criterion is satisfied, since 0000, 0011, 0101, 0110, 1001, 1010, 1100, 1111, 00010, 00100, 01000, 01110, 10001, 10111, 11011, and 11101 are selected command address codes. Similar to above, it now follows that the sets of 7-digit, 8-digit, ... , $(n-1)$-digit, and n-digit command address codes are empty since every member of each of the aforementioned sets contains a 6-digit subsequence which is identical to at least one member of the set of all 6-digit binary words. Thus, a trend is established. We conclude that if: (1) every member of one of the two sets of all j-digit binary words, which have for each set a minimum mutual Hamming distance of 2 and an upper bound of $B(j, 2)$, were used as a command address code; and (2) the code subsequences criterion were satified, there exist a set of $(j+1)$-digit binary words, which have a minimum mutual hamming distance of 2 and an upper bound of $B(j, 2)$, which are <u>usable</u> as command address codes. Now, if: (1) every member of the aforementioned set of $(j+1)$-digit binary words is used as a command address code; and (2) the code subsequences criterion is again satisfied, every binary word, whose word length is greater than $j+1$, is <u>unusable</u> as a command address code.

The non-empty sets of 4-digit, 5-digit, ... , 31-digit, and 32-digit command address codes can be increased but only at the expense of further reducing the number of codes in one or more sets. For example, suppose we select (for command address codes) one-half of the members from one of the two sets, {0000, 0011, 0101, 0110, 1001, 1010, 1100, 1111} and {0001, 0010, 0100, 0111, 1000, 1011, 1101, 1110}, of 4-digit binary words which have, for each set, a minimum mutual Hamming distance of 2. The selected 4-digit binary words (say, 0000, 0011, 0101, and 0110) constitute the selected set of 4-digit command address codes which have a minimum mutual Hamming distance of 2. Satisfying the code subsequences criterion, the sets of 5-digit, 6-digit, and 7-digit command address codes (which have, for each set, a minimum mutual Hamming distance of 2) are respectively {00010, 00100, 01000, 01110, 10001, 10111, 11010, 11100}, {010010, 011110, 101001, 111011, 111101}, and {0111110, 1111111} which indicates that the set(s) of 8-digit and/or 9-digit command address codes are/is empty. It follows that the 10-digit, 11-digit, ... , 31-digit, and 32-digit sets of command address codes are also empty.

Other Work on Command Address Codes at GSFC

Mr. Morton Foxe, Operations Evaluation Branch, has recently completed an analysis and a written report* on command address codes. In his report, Mr. Foxe predicts (based on his analysis) that, for calendar years 1966-70, NASA will have a maximum number of 12 satellites which: (1) utilize the same frequency; and (2) require command address codes. Reasoning that the same set of command address codes can be used for different frequencies, Mr. Foxe subsequently concludes that the set of 8-digit binary words, which have a minimum mutual Hamming distance of 4 and an upper bound of 16, is the optimum set of command address codes. Furthermore, he gives the Experimenters and/or Space Scientists the additional choice of using members of the set of 4-digit binary words, which have a minimum mutual Hamming distance of 2 and an upper bound of 8, as command address codes.

As might be expected from the above analysis of this report, limitations are imposed on the set of 8-digit command address codes if members of the set of 4-digit binary words are used as command address codes (i.e., the number of usable 8-digit command address codes is reduced or decreased according as the number of 4-digit command address codes is increased). In an analysis, the author of this report determined the set of 8-digit command address codes which have a minimum mutual Hamming distance of 4 after: (1) selecting as command address codes, every member of one of the two sets of all 4-digit binary words which have, for each set, a minimum mutual Hamming distance of 2; and (2) satisfying the code subsequences criterion. It was discovered that the total number of members in the set of 8-digit command address codes is reduced to one-half of its upper bound, $B(8,4)$. That is, the total number of 8-digit command address codes is 8 if conditions (1) and (2), above, are satisfied. However, the "reduced upperbound" of the members of the

*Not published at this writing.

aforementioned set of 8-digit command address codes is not unique. As a matter of fact, if we use the set of 4-digit command address codes which have a minimum mutual Hamming distance of 2 and an upper bound of 8, we may use any word length k, $k > 4$, to determine a set of k-digit command address codes which have an upperbound of 8 and a minimum mutual Hamming distance of d'', where $2 \leq d'' \leq k/2$. Also, as might be expected, every j-digit binary word ($j = 1, 2, \ldots, n$), which is not a member of the two aforementioned sets (i.e., the sets of 4-digit and k-digit command address codes, $k = 5, 6, \ldots, n$), is unusable as a command address code.

In a less exhaustive analysis, the author of this report found that the total number of members in the "optimum" set of 8-digit command address codes was respectively reduced (from an upper bound of 16) to 12, 13, and 14 if: (1) 4 "selected" members, 2 "selected" members, and 1 "selected" member of one of the two sets of all 4-digit binary words (which have, for each set, a minimum mutual Hamming distance of 2) are respectively used as command address codes; and (2) the code subsequences criterion is satisfied. In Table I, the "reduced" sets of 8-digit command address codes are shown along with the "selected" 4-digit command address codes.

CONCLUSIONS AND FINAL REMARKS

For a set of command address codes of several word lengths, say i, $i + 1, i + 2, \ldots, n$ for $i = 1, 2, \ldots$, the number of usable command address codes, after the code subsequences criterion is satisfied, is or is about (in most cases) equivalent to the upper bound for the number of members in the set of all i-digit binary words or codes. Depending on the number of i-digit command address codes used, a total number of at least $B(i, 1)$ or 2^i command address codes can be dispersed among the word lengths $i, i + 1, \ldots, n - 1$, and n. For example, we can have:

(1) $B(i, 1)$ or 2^i i-digit command address codes, and no $(i + 1)$-digit, $(i + 2)$-digit, . . . , $(n - 1)$-digit, or n-digit command address codes

(2) $B(i, 2)$ or 2^{i-1} i-digit command address codes, and $B(i, 2)$ or 2^{i-1} $(i+1)$-digit command address codes with no $(i + 2)$-digit, $(i + 3)$-digit, ... , $(n - 1)$-digit, or n-digit command address codes

. . . .

(m) at least one command address code of word lengths $i, i + 1, \ldots, n-1$, and n, for $i > 1$ (i.e., at least one i-digit command address code; at least one $(i + 1)$-digit command address code; etc. . . .).

It is clear that the greater the number of command address codes of the same word length, the smaller the number of distinct word lengths for which command address codes can be used. Expressing this mathematically, let:

(1) N be the number of command address codes of the same word length, say i, and

(2) S be the number of distinct word lengths for which command address codes can be used, then

$$\lim_{N \to 2^i} S = 1$$

Furthermore, if we wish to use a finite number of word lengths, say i, i + 1, i + 2, ... , for which command address codes can be used, it is necessary to: (1) choose i sufficiently large so that 2^i is equal to or greater than the desired total number of command address codes of word lengths i, i + 1, i + 2, ... ; and (2) select the command address codes in a manner so that they are "properly" dispersed among the word lengths i, i + 1, i + 2, ... This appears, to the writer, to create a tedious task for the GSFC Frequency Control Officer in future assignments of command address codes to NASA spacecraft. For once a finite number of word lengths, for which command address codes can be used, are made available to the Experimenters and/or Space Scientists, the requests and demands for command address codes of a particular word length may become excessive, and thereby cause the command address codes of other word lengths to be adversely affected.

RECOMMENDATIONS

In view of the above analysis and conclusions, it is recommended that one word length be used for all of the command address codes. The use of one word length precludes the use or application of the code subsequences criterion. This will simplify, to a great degree, future assignments of command address codes by the GSFC Frequency Control Officer.

The word length should be: (1) large enough to provide a sufficient number of codes for a chosen Hamming distance; and (2) small enough to alleviate any unnecessary handling of bits. The upper bound, B(n, d), for the number of codes of length n which have a minimum mutual Hamming distance of d has been computed by the FAP computer program, and is given in Table II for n = 4, 5, . . . , 13 and d = 2, 3, ... , 13. (NOTE: the reader is invited to compare Table II with the theoretical predictions).

Command address codes of two distinct word lengths can be assigned without too much difficulty even though they are interrelated. Hence, as a second choice, the author also recommends two distinct word lengths.

Table I

Optimum Sets of 4-Digit and 8-Digit Command Address Codes which Respectively have a Minimum Mutual Hamming Distance of 2 and 4.

4-Digit Command Address Codes (Binary)	8-Digit Command Address Codes (Binary)
0000	00010001
0011	00100010
0101	01000100
0110	01110111
1001	10001000
1010	10111011
1100	11011101
1111	11101110

4-Digit Command Address Codes (Binary)	8-Digit Command Address Codes (Binary)
0000	00010001
0011	00100010
1100	00101101
1111	01000100
	01001011
	01110111
	10001000
	10110100
	10111011
	11010010
	11011101
	11101110

4-Digit Command Address Codes (Binary)	8-Digit Command Address Codes (Binary)
0000	00010001
0011	00100010
	00101101
	01000100
	01001011
	01110111
	01111000
	10001000
	10110100
	10111011
	11010010
	11011101
	11101110

Table I (Continued)

4-Digit Command Address Codes (Binary)	8-Digit Command Address Codes (Binary)
0000	00010001
	00011110
	00100010
	00101101
	01000100
	01001011
	01110111
	01111000
	10001000
	10110100
	10111011
	11010010
	11011101
	11101110

Table II

The Upper Bound for the Number of Binary Words or Codes of Length n which have a Minimum Mutual Hamming Distance of d.

	d											
n	2	3	4	5	6	7	8	9	10	11	12	13
4	8	2	2	0	0	0	0	0	0	0	0	0
5	16	4	2	2	0	0	0	0	0	0	0	0
6	32	8	4	2	2	0	0	0	0	0	0	0
7	64	16	8	2	2	2	0	0	0	0	0	0
8	128	16	16	4	2	2	2	0	0	0	0	0
9	256	32	16	4	4	2	2	2	0	0	0	0
10	512	64	32	8	4	2	2	2	2	0	0	0
11	1024	128	64	16	8	4	2	2	2	2	0	0
12	2048	256	128	16	16	4	4	2	2	2	2	0
13	4096	512	256	32	16	8	4	2	2	2	2	2

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